# Comparative Effectiveness of Melin Integral Transform and Harmonized Dimensional Integral Transform in Capital Market Efficiencies

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***ABSTRACT***

*Melin transform is an integral transform which has been proposed for many types of physical and economical system even in perpetual option pricing. In financial market price stability, we observed that it is weakly efficient. We herein presented harmonized dimensional integral transform as a generalized state of the melin transform with the aim of testing for market efficiency. The effect of our approach is that, the technique is stable and a strongly efficient model for capital market, having a significant impact on returns and taking account of issues of positive bias natures of investors which the melin integral transform ignores. In addition, the model allows one to define an optimal positioning between a reduction in market and likelihood of a bankruptcy within the given time horizon.*

***Key Words:****Melin Transform, Harmonized transform, Capital Market efficiency, Normal Gussianand RaydonNicodyn.* ***Academic Discipline:*** *Financial Mathematics.*

## 1.1 INTRODUCTION

Efficiency refers only to historical information which is contained in every private trading agent's information set. In a financial market system, there are several reasons for using a stable model to describe an efficient market system. The first is where there are solid theoretical reasons for expecting a non Gaussian model. Example, reflection of a rotating mirror yielding a Cauchy distribution, hitting times for a Brownian motion yielding a levy distribution, the gravitation field of stars yielding the Holtsmark distribution (Barndorff-Nielsen et al 2000/2001). The second reason is empirical because many large data sets exhibit heavy tails and skewness. The strong empirical evidence for these features is used by many to justify the use of stable and efficient models. Examples in finance and economics are given in Mandelbrot 1965, Esser (2003).McKean and Fama (1965), Franke et al (2004) and Carr et al (2000). Such data sets are poorly describe by Normal Gaussian model, but can be well describe by a stable distribution. The Melin transform has been successful in perpetual option pricing theory both in physical and economic system but the applicability to problem in modern finance theory have not been studied extensively yet suggested by Panini and Srivastav (2004). To this, it is observed that in complete financial market efficiency, it has no significant on interest rate fluctuation. In practice, random fluctuation of interest rate have a significant contribution to change of returns and even though interest rate fluctuate randomly in market, many models do not fully consider their bias natures owing to their generally limited impact on returns. However, in this paper, we show that harmonized dimension transform technique is an efficient financial market model having a significant impact on price returns as it takes account of issue of positive bias natures of investors.

## 2.1 EFFICIENT AND INEFFICIENT MARKET

According to Fama (1970), financial market systems are separated into standard and exotic market. Standard market is subject to certain regularity, standardization conditions and organized exchange while exotic have more complicated payoff traded in the over- the-counted-market and the leading players are the commercial and investment banks. The more complicated payoff, however, introduces much greater complexity to the valuation problem, thus demanding sophisticated mathematical machinery.
Efficiency refers to when the relevant information set is reflected in the market price and weakly efficient implies that successive price changes or returns are independently distributed. Formally, let the market model be described by a probability triple $(\Omega, \mathcal{F}, \mathbb{P})$. Suppose that trading takes place in continuous time, and that there is one risky security. Let $h > 0, t \geq 0$ *and let* $r_h(t+h)$ denote the return of security from t to t + h, where h is a fixed time lag and let $S(t)$ be the price of the risky security at time t. also let $\mathscr{F}$(t) be the collection of historical information available to every market participant at time t.

Then the market is **weakly efficient** based on the following axioms

**AXIOM 1:**

i) $if \quad x\mathscr{F}(t)$ Exist $\forall\, x \in R.$

ii) $\mathbb{P}[(r_h(t+h) \leq x\mathcal{F}(t)] \qquad \forall\, x \in R. h > 0, t \geq 0.$

Here, the information $\mathscr{F}$(t) which is publicly available at time t is nothing other than the generated $\sigma - algebrae$ of the price returns with the random variable $x$.

Where $\quad r_h(t+h) = log\frac{S(t+h)}{S(t)}$ is the rate of return.

On the other hand, a market is strongly efficient if

**AXIOM 2:**

$i)$ $if \quad x\mathscr{F}(t)$ Exist $\forall \; x \in R$

ii) $\mathbb{P}(r_h(t+h) \geq x\mathcal{F}(t)] \qquad \forall\, x \in R.\; h > 0, t \geq 0.$

Note that Dimension is defined as $\lim_{size\to 0}\frac{\log\ bulk}{\log\ size}$ which is equivalent to rate of return

$$r_h(t+h) = log\frac{S(t+h)}{S(t)}.$$

## 2.2 FINANCIAL INSTRUMENTS

Financial instruments are defined in terms of other underlying quantities such as stock, indices, currency and interest rates or volatilities. Here $S_t = St$ is the price of underlying asset, subscripts are used to emphasize on evolutions of underlying asset process through time and due to the time dependency, $h = T - t$ is the remaining time of maturity. $S_T$ is normally distributed $(X\sim N(\mu,\sigma^2))$ if it has a density

Normal or Gaussian distribution $X\sim N(\mu,\sigma^2)$ if it has a density

$$S_T = \frac{1}{\sqrt{2\pi\sigma}}e^{\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)} - \infty < x < \infty \quad \Rightarrow S(x,\mu,\sigma,\pi). \tag{1}$$

$$= \frac{1}{\sqrt{2\pi\sigma}}e^{\left(-\frac{(InS_t)^2}{4\theta}\right)}.$$

Where, $\ln S_t = x - \mu$ $\quad and\ \ \theta = 2\ \ where\ \ \sigma^2 = 4.$ Take $S_T = F(x)$.

The cumulative distribution function for which there is a close form expression is $F(x) = P(X \le x) = where\ \Phi(z) =$ probability that a standard normal random variable is less than or equal to z.

For a given pay off function$g : R^+ \to R$ written as $S$ can be regarded as a financial instrument that pays the holder $g(ST)$ at expiry T for all $S \in R^+$ then we take $f(x)\ to\ be$

$$S_T = S_t e^{\left(\left(\mu-q-\frac{1}{2}\sigma^2\right)(T-t)+\sigma(W_T-W_t)\right)} for\ all\ t \in [0.T], \tag{2}$$

$Where\ \ (T-t)\ = \ \text{hand}W_T - \ W_t\ = \beta,$

so that equation (2) reduces to

$$S_T = S_t e^{\left(\left(\mu-q-\frac{1}{2}\sigma^2\right)h+\sigma(\beta)\right)} for\ all\ t \in [0.T] \quad \Rightarrow S(\mu,\sigma.q,h,\beta). \tag{3a}$$

Here,$S(t)$ is the price of the risky security at time t , $\mu$ is the appreciation rate of price or riskless interest rate, $q$ is the dividend yield, $\beta$ is one dimensional Brownian motion and $\sigma > 0$ is the volatility and it is well known that the logarithm of S grows linearly in the long- run .

The RadonNicodyn derivative given as

$$S_T = e^{-\left(\frac{1}{2}\frac{(\mu-q)^2}{\sigma^2}T - \frac{\mu-q}{\sigma}\beta\right)}\ for\ all\ t \in [\ 0,T] \Rightarrow S(\mu,q,\sigma.\beta,T). \tag{3b}$$

## 3.1 THE MELIN TRANSFORM

In mathematics, the Melin transform is an integral transform that may be regarded as the multiplicative version of the two-sided laplace transform closely related to Dirichlet Series, Fourier transform, theory of Gamma and Allied special Function and often used in number theory, Mathematical Statistics, and theory of asymptotic expansions.

Robert Hjalmar Melin (1854-1933) gave his name to the Melin transform that associates to the locally Lebesgue integrable function $f(x)$ defined over positive real number the complex function $M(f(x),w)$ defined by

$$\mathcal{F}(t) = M(f(x),w) = f(w) = \int_0^\infty f(x)x^{w-1}dx, \quad (4a)$$

Its probability density function is $F(x) = \frac{1}{\gamma(r)}X^{r-1}e^{-x}, r > 0.$

Where $X^{r-1}$ is the Melin kernel and the expectation of the continuous random variable $X$ is given by

$$E[X] = \int_{-\infty}^{\infty} XF(x)dx = 1. \quad (4b)$$

$$\int_0^\infty F(x)\,dx = \int_0^\infty \frac{1}{\gamma(r)}X^{r-1}e^{-x}dx = \frac{1}{\gamma(r)}\int_0^\infty X^{r-1}e^{-x} = \frac{\gamma(r)}{\gamma(r)} = 1. \quad (4c)$$

The Melin transform is defined on a vertical strip in the w-plane, whose boundaries are determined by the asymptotic behavior of $f(x) as\ x \to 0^+\ and\ x \to \infty\ the\ largest\ srip\ (a,b)$ in which the integral converges is called the fundamental strip. The condition

$$f(x) = O(x^u) \quad for \quad x \to 0^+$$

And
$$f(x) = O(x^v) \quad for \quad x \to \infty$$

When $u > v$, guarantee the existence of $M(f(x),w)$ in the strip $(-u,-v)$. Thus, the existence is granted for locally integrable functions, whose exponent in the order of 0 is strictly larger than the exponent of order at infinity.

Conversely, $if\ F(x)$ is an integrable function with fundamental strip $(a,b)$, then if c is such that $a < c < b\ and\ f(c+it)$ is integrable, the equality

$$\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \tilde{f}(x)x^{-w}dw = f(x), \quad (4d)$$

Holds almosteverywhere. Moreover, if $f(x)$ is continuous, then the equality holds everywhere on $(0,\infty)$. Simple changes of variables in definition of the Melin transforms yield to a whole set of transformation rules and facilitates the computations. In particular, $if\ f(x)$ admits the Melin transform on the strip $(a,b)$ and $\alpha,\beta$ are positive reals, then the following relation holds.

$$M(f(\alpha x),w) = \alpha^{-w}\tilde{f}(x) \quad on \quad (a,b).$$

$$M(\,f(x^{\alpha}),w) = \tilde{f}(w+\alpha) \qquad on \qquad (a,b).$$

$$M(\,f(x^{\alpha}),w) = \frac{1}{\alpha}\tilde{f}(w/\alpha), \alpha > 0 \; on \quad (a\alpha, b\alpha).$$

$$M\left(f\left(\frac{1}{x}\right), w\right) = -\tilde{f}(-w) \quad on \;\; (-b,-a).$$

$$M\big(x^{\beta}\, f(x^{\alpha}),w\big) = \frac{1}{\alpha}\tilde{f}(w+\beta/\alpha), \qquad \alpha > 0 \quad on \;\; (a\alpha, b\alpha).$$

$$M\left(x\frac{d}{dx}\, f(x),w\right) = -\tilde{f}(w), \qquad on \;\; (a^{*}, b^{*}).$$

$$M\left(\frac{d}{dx}\, f(x),w\right) = -(w-1)\tilde{f}(w-1), \qquad on \;\; (a^{*}-1,\; b^{*}-1).$$

$$M\left(\frac{d^{n}}{dx^{n}}\, f(x),w\right) = (-1)^{n}\frac{\Gamma(w)s}{\Gamma(w-n)}\tilde{f}(w-n), \qquad on \;\; (a^{*}-n,\; b^{*}-n).$$

For a proof of some of these relations we refer to Titchmarsh (1986) and Sneddon (1972). The change of variables $x = e^{s}$ shows that the Melin transform is closely related to the Laplace- Fourier version. In particular, if $F(f(x(,w) and\; L(f(x),w)$ denote the two sided Fourier and Laplace transform, respectively, then we have

$$M(\,f(x),w) = L(\,f(e^{-x}),w) = F(\,f(e^{-x}),-iw). \qquad (5)$$

However, there are numerous applications where it proves to be more convenient to operate directly with the Melin rather than the Laplace and Fourier transform.

## 4.0 APPLICATION

According to Robert and Rainer (2008) application of Melin transform using the Gaussian distribution technique and the Radon Nikodyn derivatives, from their application it is confirmed that the result does represent an efficient market in the weak sense. The result is thus.

Given $\breve{\Phi}(w) = x\mathcal{F}(t)$ as the melin transform of $\emptyset(S) = f(x)$ were

$$\emptyset(S) = \frac{1}{\sqrt{2\pi\sigma}}e^{\left(-\frac{(Ins)^{2}}{4\theta}\right)} \quad and$$

$$S_{T} = \frac{1}{\sqrt{2\pi\sigma}}e^{\left(-\frac{(x-\mu)^{2}}{2\sigma^{2}}\right)}$$ from equation (1).

Here, $Ins = x - \mu \quad and \;\; \theta = 2 \;\; where \;\; \sigma^{2} = 4.$

$\breve{\Phi}(w) = \int_{0}^{\infty} \emptyset(S)\; S^{w-1} dS$ from equation (4).

From there result, using the transformation in Erdelyi et. al. (1954).

$\breve{\Phi}(w) = \int_0^\infty \frac{1}{\sqrt{2\pi\sigma}} e^{\left(-\frac{(Ins)^2}{4\theta}\right)} S^{w-1} dS \qquad Re(\theta) \geq 0.$

The result yield $\frac{S^{\beta_2}}{n\sigma\sqrt{2\pi(T-t)}} e^{-\frac{1}{2}(\frac{Ins}{n\sigma\sqrt{T-t}})^2}$.

**TESTING FOR EFFICIENCY**: we can see that from the result

$$\mathscr{F}(\mathrm{t}) = \frac{S^{\beta_2}}{n\sigma\sqrt{2\pi(T-t)}} e^{-\frac{1}{2}(\frac{Ins}{n\sigma\sqrt{T-t}})^2} \tag{6}$$

And

$$r_h(t+h) = \ log\frac{S(t+h)}{S(t)} = -\frac{(Ins)^2}{4\theta} \tag{7}$$

$-\frac{(Ins)^2}{4\theta} \leq x(\frac{S^{\beta_2}}{n\sigma\sqrt{2\pi(T-t)}} e^{-\frac{1}{2}(\frac{Ins}{n\sigma\sqrt{T-t}})^2})$. Axiom1 satisfied

**4.1 From equation (3a) and (4a) we have**

$S_T = S_t e^{((\mu-q-\frac{1}{2}\sigma^2)h+\sigma(\beta))} for\ all\ t \epsilon [0.T] \Rightarrow S(\mu, \sigma. q, h, \beta).$ From equation (3a).

Where $f(X) = S_t e^{((\mu-q-\frac{1}{2}\sigma^2)h+\sigma(\beta))}$. Equation (4a).

Let $x = ((\mu - q - \frac{1}{2}\sigma^2) h + \sigma(\beta))$, then $f(X) = S_t e^x$.

Then equation (4a) gives

$$\mathcal{F}(t) = \int_0^\infty f(X)\, X^{w-1} dx$$

$$= \int_0^\infty S_t e^x\, X^{w-1} dx$$

$$= S_t \int_0^\infty e^x\, X^{w-1} dx$$

$$\int_b^a v du = [uv]_b^a - \int_b^a v du$$

$$\mathscr{F}(t) \quad = \frac{S_t\ Tan\frac{\pi}{2}((\mu-q-\frac{1}{2}\sigma^2)h+\sigma(\beta))}{((\mu-q-\frac{1}{2}\sigma^2)h+\sigma(\beta))(w-1)}. \tag{8}$$

$$r_h(t+h) = \ log\frac{S(t+h)}{S(t)} = ((\mu - q - \frac{1}{2}\sigma^2) h + \sigma(\beta)). \tag{9}$$

Here, the information $\mathscr{F}$(t) which is publicly available at time t is nothing other than the generated $\sigma - algebrae$ of the price returns .

We have $((\mu - q - \frac{1}{2}\sigma^2)h + \sigma(\beta)) \leq x\ (\frac{S_t\, Tan\frac{\pi}{2}((\mu-q-\frac{1}{2}\sigma^2)h+\sigma(\beta))}{((\mu-q-\frac{1}{2}\sigma^2)h+\sigma(\beta))(w-1)})$. Axiom 1 satisfied

**4.2 Using equation (3b) and (4a) we have**

$S_T = e^{-\left(\frac{1}{2}\frac{(\mu-q)^2}{\sigma^2}T - \frac{\mu-q}{\sigma}\beta\right)} for\ all\ t\ \epsilon\ [0.T] \Rightarrow S(\mu, \sigma, q, \beta, T)$ from equation (3b),

$$f(X) = e^{-\left(\frac{1}{2}\frac{(\mu-q)^2}{\sigma^2}T - \frac{\mu-q}{\sigma}\beta\right)}$$

Let $x = \left(\frac{1}{2}\frac{(\mu-q)^2}{\sigma^2}T - \frac{\mu-q}{\sigma}\beta\right)$

$\mathcal{F}(t) = \int_0^\infty f(X)\ X^{w-1}dx$ from equation (4a),

$$= \int_0^\infty e^{-x}\ X^{w-1}dx$$

$$\int_b^a vdu = [uv]_b^a - \int_b^a vdu$$

$$\mathscr{F}(t) = \frac{Tan\frac{\pi}{2}\left(\frac{1}{2}\frac{(\mu-q)^2}{\sigma^2}T - \frac{\mu-q}{\sigma}\beta\right)}{\left(\frac{1}{2}\frac{(\mu-q)^2}{\sigma^2}T - \frac{\mu-q}{\sigma}\beta\right)(w-1)}. \quad (10)$$

$$r_h(t+h) = log\frac{S(t+h)}{S(t)} = -\left(\frac{1}{2}\frac{(\mu-q)^2}{\sigma^2}T - \frac{\mu-q}{\sigma}\beta\right)\ . \quad (11)$$

We have that$-\left(\frac{1}{2}\frac{(\mu-q)^2}{\sigma^2}T - \frac{\mu-q}{\sigma}\beta\right) \leq x\ (\frac{Tan\frac{\pi}{2}\left(\frac{1}{2}\frac{(\mu-q)^2}{\sigma^2}T - \frac{\mu-q}{\sigma}\beta\right)}{\left(\frac{1}{2}\frac{(\mu-q)^2}{\sigma^2}T - \frac{\mu-q}{\sigma}\beta\right)(w-1)})$. Axiom 1 satisfied.

**5.1 THE HARMONIZED DIMENSIONAL TRANSFORM**

According to Ying and Lawrence 2008, the average fractal dimension is the $P^{th}$ fractal distribution andThe basis of fractal geometry is the idea of self similarity hence fractal dimension average all over the fractal properties. It is a natural dimensional measure relevant for dynamic system given as

$\mu(\beta) = \lim_{T\to\infty}\frac{1}{T}\int_o^T I_\beta(\varphi_t(X_o))\,dt \Rightarrow \frac{1}{\Delta\alpha}\int_{min}^{max} F(\alpha)\,d\alpha$ ( Zhang, 2011), (12)

Harmonized as

$\mathcal{F}(t) = \lim_{T\to\infty} \frac{1}{T}\int_o^T I_\beta(\varphi_t(X_o))^\lambda dt \Rightarrow \mathscr{F}(t) = \frac{1}{\Delta\alpha}\int_0^\infty (f(x))^\gamma\, dx$(Osu and Ogwo, 2015). (13)

The evolution operator $\varphi_t$, tells how the state of the system changes with time and it is a family of function $\varphi_t : R^m \to R^m$ that maps the current state of the system into the future state at a time units later and $\varphi_t$ $satisfy$ $\varphi_0(X) = X$ $and$ $\varphi_{t+s}(X) = \varphi_t(\varphi_s(X))$ .
$\varphi_t$ Can be defined either as a discrete map or in terms of ordinary differential equation.

- $\alpha \epsilon (0,1]$ is the extent of short selling in the market or "going long". Here an increased $\alpha$ is associated with an increased tendency to short sell or going long. In this model, we presume that the returns evolve according to the strength of the various agents trading in the market at any given time, each agent determines a threshold which signals whether the market is overbought or oversold and the agent becomes more risk averse in there trading strategies when these overbought or oversold threshold are breached.

- $\gamma$ is the maximum displacement parameter which can be defined as the market bias from the unbiased value. Our assumption $\gamma$means that the presence of the bias increases the average size of the departure of returns from the trend growth rate. Here, $\mathscr{F}$ is assumed continuous and odd on $R$ so that the positive and negative returns are treated symmetrically. Moreover, in order to make the bias be modest, we require $\lim_{|x|\to\infty} x\mathcal{F}(x) \to R \,\epsilon\, (0,T]$ $or$ $(0,\frac{\sigma^2}{2}]$. Moreover; we assume that the investors can estimate the value of $\sigma$ by tracking the size of any large deviations.

**5.2 Using equation (1) and (13) we have**

$$\mathcal{F}(t) = \frac{1}{\Delta\alpha}\int_0^\infty (f(X))^\gamma\, dx$$

$$= \frac{1}{\Delta\alpha}\int_0^\infty \left(\frac{1}{\sqrt{2\pi\sigma}} e^{\left(-\frac{(Ins)^2}{4\theta}\right)}\right)^\gamma dx$$

If $x = \frac{(Ins)^2}{4\theta}$ then $\mathcal{F}(t) = \frac{1}{\Delta\alpha}\int_0^\infty \left(\frac{1}{\sqrt{2\pi\sigma}} e^{-x}\right)^\gamma dx$

$$\mathcal{F}(t) = \frac{1}{\gamma(\sqrt{2\pi\sigma})^\gamma \Delta\alpha} x^{1-\gamma}. \qquad (14)$$

**Then testing for efficiency** we can see that

$\mathscr{F}$(t) $= \frac{1}{\gamma(\sqrt{2\pi\sigma})^\gamma \Delta\alpha} x^{1-\gamma}$ a.s on $\gamma \leq 0$.

$= \frac{x^{1-\gamma}}{\gamma(\sqrt{2\pi\sigma})^{\gamma}\Delta\alpha} \quad \Rightarrow \quad \frac{(\frac{(Ins)^2}{4\theta})^{1-\gamma}}{\gamma(\sqrt{2\pi\sigma})^{\gamma}\Delta\alpha}$ from (8),

$r_h(t+h) = \ log\frac{S(t+h)}{S(t)} = -\frac{(Ins)^2}{4\theta}$ from (7).

Hence $-\frac{(Ins)^2}{4\theta} \geq \ x\left(\frac{(\frac{(Ins)^2}{4\theta})^{1-\gamma}}{\gamma(\sqrt{2\pi\sigma})^{\gamma}\Delta\alpha}\right) \quad a.s$ on the event $\gamma \leq 0$. axiom 2 satisfied.

Here, the information $\mathscr{F}$(t) which is publicly available at time t is nothing other than the generated $\sigma - algebrae$ of the price returns .

**5.3 Using Equation (3a) and (13) gives**

$S_T = S_t e^{((\mu-q-\frac{1}{2}\sigma^2)h+\sigma(\beta))} for\ all\ t\ \epsilon\ [0.T] \Rightarrow S(\mu,\sigma.q,h,\beta)$. From equation (3a),

$f(X) = S_t e^{((\mu-q-\frac{1}{2}\sigma^2)h+\sigma(\beta))}$ in equation (5),

Let $x = \left(\left(\mu - q - \frac{1}{2}\sigma^2\right)h + \sigma(\beta)\right)$, then equation (5) gives

$$\mathcal{F}(t) = \frac{1}{\Delta\alpha}\int_0^{\infty}(S_t e^x)^{\gamma}dx$$

$$\mathcal{F}(t) \quad = \quad -\frac{(S_t)^{\gamma}}{\gamma x^{\gamma-1}\Delta\alpha}. \tag{15}$$

$= - \frac{(S_t)^{\gamma}}{\gamma((\mu-q-\frac{1}{2}\sigma^2)h+\sigma(\beta))^{\gamma-1}\Delta\alpha}$ from (15),

$$r_h(t+h) = \ log\frac{S(t+h)}{S(t)} = \left(\left(\mu - q - \frac{1}{2}\sigma^2\right)h + \sigma(\beta)\right). \tag{16}$$

Hence $\left(\left(\mu - q - \frac{1}{2}\sigma^2\right)h + \sigma(\beta)\right) \geq -x\frac{(st)^{\gamma}}{\gamma((\mu-q-\frac{1}{2}\sigma^2)h+\sigma(\beta))^{\gamma-1}\Delta\alpha} \quad a.s.$ axiom 2 satisfied

**5.4 Using Equation(3b) and (13) we have**

$ST = e^{-\left(\frac{1}{2}\frac{(\mu-q)^2}{\sigma^2}T-\frac{\mu-q}{\sigma}\beta\right)} for\ all\ t\ \epsilon\ [0.T] \Rightarrow S(\mu,\sigma.q,\beta,T)$ from equation (3b),

$f(X) = e^{-\left(\frac{1}{2}\frac{(\mu-q)^2}{\sigma^2}T-\frac{\mu-q}{\sigma}\beta\right)}$ in equation (5),

and $x = \left(\frac{1}{2}\frac{(\mu-q)^2}{\sigma^2}T - \frac{\mu-q}{\sigma}\beta\right)$. Equation (5) gives

$$\mathcal{F}(t) = \frac{1}{\Delta\alpha}\int_0^\infty (e^{-x})^\gamma dx$$

$$\mathcal{F}(t) \;=\; \frac{1}{\gamma x^{\gamma-1}\,\Delta\alpha}.(15)$$

$$= \frac{1}{\gamma\left(\frac{1}{2}\frac{(\mu-q)^2}{\sigma^2}T-\frac{\mu-q}{\sigma}\beta\right)^{\gamma-1}\Delta\alpha}\text{from(15)},$$

$$r_h(t+h) = \; log\frac{S(t+h)}{S(t)} \;=-\left(\frac{1}{2}\frac{(\mu-q)^2}{\sigma^2}T-\frac{\mu-q}{\sigma}\beta\right)\;. \qquad (16)$$

Hence $-\left(\frac{1}{2}\frac{(\mu-q)^2}{\sigma^2}T-\frac{\mu-q}{\sigma}\beta\right) \geq \; x\,\frac{1}{\gamma\left(\frac{1}{2}\frac{(\mu-q)^2}{\sigma^2}T-\frac{\mu-q}{\sigma}\beta\right)^{\gamma-1}\Delta\alpha}$ $a.s$ on the event $\gamma \leq 0$. Axiom 2 satisfied.

### 5.5 Harmonized dimensional integral transform as a generalized state of Melin integral transform

Two measures are said to be equivalent when they are equal to zero but if they have the same dimension then it is fractal. (Falcon ,1985).

$$\mathcal{F}(t) = \frac{1}{\Delta\alpha}\int_0^\infty (f(X))^\gamma\;dx = \;\int_0^\infty (f(X))^\gamma(\Delta\alpha)^{-1}dx \;\;\Rightarrow\;\; \int_0^\infty f(x)\;x^{w-1}dx. \qquad (17)$$

If $x = \Delta\alpha, w = 0 \;\; and\, \gamma = 1.$

## CONCLUSION

Finally we confirm that the Melin integral transform does represent an inefficient market in the weak sense. The presence of the indicator function $\gamma$ in the harmonized dimensional integral transform indicates that, the stronger the bias nature of investors, the larger the average values of prices excursions and consequently the smaller the volatility that arises around the average values. This persistence could make the investors believe that the cumulative returns are close to their true values and are unbiased, the agents become more risk averse in there trading strategies when their overbought or oversold threshold are breached. Hence, allowing one to define an optimal positioning between a reduction in market and likelihood of a bankruptcy, addressing the issue of inefficiency and making the model strongly efficient.

**REFERENCES**

Barndorff-Nielsen ,O.E., Mikosch,T. and Resnick, S. (Eds.) (2001) Levy Processes – Theory and Application. Boston: Birkhauser.

Barndorff-Nielsen ,O.E.,andShephard,N.(2000) Modelling by Levy Processes for financial Econometrics. In O.E. Barndorff-Nielsen ,O.E., Mikosch,T. and Resnick, S. (Eds.) (2001) Levy Processes – Theory and Application. Boston: Birkhauser, 283-318.

B. B.Mandelbrot, The Variation Of Certain Speculative Prices. Journal OfBusiness XXXVI (1965) 392-417.

Carr, P., Geman, H., Madan, D.H. And Yor, M. (2000).*The Fine Structure Of Asset Returns: An EmpiricalInvestigations*. **Preprint**.

Esser,A. 2003, General Valuation Principle for Arbitrary Payoffs and Applications to Power Options Under *Stochastic Volatility, Financial Markets and Portfolio Management* **17**, 351-372

E.F. Fama. (1970). Efficient capital markets: a review of theory and empirical work, J. Finance, 2,25, 1970.

Falconer, k.J. (1985).*The Geometry of Fractal Sets.Cambridge University Press.*

J. Franke, W. Hardle, and C. M. Hafner, *Statistics Of Finance Markets Springer*, 2004.

Lin Zhang. Multifractal Properties of the industry Indices for Chinese and Japanese Stock Markets. 2011 3rd international conference on information and financial Engineering IPEDR vol.12(2011) LACSIT Press, Singapore.

Mckean, H. 1965, Appendix: A Free Boundary Problem For Heat Equation Arising From A Problem In Mathematical Economics, *Industrial Management Review* **6**.32-39.

Panini, R. and Srivastav, R. : (2004), Option Pricing With Melin Transforms, *Mathematical and Computer Modelling* ***40****, 43-56.*

Robert .F. and Rainer.S. (2008).pricing American Options with Melin Transforms. Mathematical Subject Classification (2000): 44A15, 60H30.

Sneddon,**I**. : (1972), *The Use of Integral Transforms*, Istedn, McGraw-Hill, New York.

Titchmarsh, E.: (1986), *Introduction to the Theory of Fourier Integrals*, 2ndedn, Chelsea Publishing Company.

YingjieLan and Lawrence M. Leemis,. The Logistic-Exponential Survival Distribution, Wiley Interscience Virginia(2008), 23187-8795.